

Lepton flavor violating tau decays in type-III seesaw mechanism

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Abstract

In this paper, the lepton flavor violating $\tau \rightarrow \ell P(V)$ ($P, V = \pi^0, \eta, \eta', \rho^0, \omega, \phi$) and $\tau \rightarrow 3\ell$ ($\ell = e, \mu$) decays are studied in the framework of the type-III seesaw model, in which new triplet fermions with a zero hypercharge ($Y = 0$) interact with ordinary lepton doublets via Yukawa couplings, and affect tree-level leptonic Z-boson couplings. We investigate the experimental bound from the leptonic Z decay to get constraint on the existing parameters space. We predict that the upper limits on the branching ratios of $\tau \rightarrow \ell P(V)$ and $\tau \rightarrow 3\ell$ can reach the experimental current limits.

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I. INTRODUCTION

In the Standard Model (SM) with massless neutrinos, lepton flavor is conserved. However, the current neutrino oscillation data experiments indicate with very convincing evidence that neutrinos are massive and lepton flavor are mixed [1]. This is a powerful incentive for considering new particles and interactions those of the Standard Model (SM) of quarks and leptons. If the neutrino oscillation phenomenon takes place actually, lepton flavor symmetry would be broken. In that case, however, lepton flavor violating (LFV) processes are still highly suppressed because of the smallness of neutrinos masses. Hence, any experimental signal of charged LFV would be a clear indication of physics beyond the SM. This fact has led to a great amount of theoretical effort for revealing the underlying new physics in the leptonic flavor sector.

LFV appears in various extensions of the SM. In particular LFV decays $\tau \rightarrow 3\ell$ (where $\ell = e$ or μ) are discussed in various models [2–5]. Some of these models with certain combinations of parameters predict that the branching fractions for $\tau \rightarrow 3\ell$ can be as high as 10^{-7} , which is already accessible in high-statistics B factory experiments. Searches for LFV in charged lepton sector such as $\tau \rightarrow \mu P(V)$ decays with a pseudoscalar or vector meson are also discussed in models with Higgs mediated LFV processes [6, 7], heavy singlet Dirac neutrinos [8], dimension-six effective fermionic operators that induce $\tau - \mu$ mixing [9], R-parity violation in SUSY [10], type III two-Higgs doublet models [10] and flavor changing Z' bosons [10]. At the LHC, the τ leptons are produced predominantly from decays of B and D mesons and W and Z bosons. In the low-luminosity phase, corresponding to an integrated luminosity of 10 fb^{-1} per year, one expects approximately 10^{12} and 10^8 τ leptons produced per year from heavy meson and weak boson decays, respectively [11]. If we restrict the τ from weak bosons decays only, and assuming that a branching ratio close to the current upper limit, we can expect approximately 10 events within the acceptance range of a typical LHC general purpose detector after one year of low-luminosity running. With 30 fb^{-1} of data, it should be possible to probe branching ratios down to a level of $\text{Br}(\tau \rightarrow 3\ell, \ell P, \ell V) \approx 10^{-8}$ at the LHC.

In order to give mass to the neutrinos, several ways have been studied in [12–14, 17]. An alternative, equally valid and rather economical possibility is to extend the lepton sector of the SM by a heavy triplet fermions and allow them to interact with the ordinary lepton

doublets via Yukawa couplings. In this scenario, the Higgs sector is unmodified, and a set of self-conjugate $SU(2)_L$ triplets of exotic leptons with zero hypercharge are added that model so-called type III seesaw mechanism.

The model has many interesting features, including the possibility of having low seesaw scale of order a TeV to realize leptogenesis [16] and detectable effects at LHC[18, 19] due to the fact that the heavy triplet leptons have gauge interactions being non-trivial under the $SU(2)_L$ gauge group. In particular, if kinematically accessible, the charged component of the triplet will be produced in high energy collision, and its decay into Higgs and light lepton[18] provides a rather spectacular signature.

Fermionic triplet effects have been studied in the lepton sector [20] such as $\tau \rightarrow 3\ell$, $\ell \rightarrow \ell'\gamma$, $Z \rightarrow \ell\ell'$, $\mu - e$ conversion and the anomalous magnetic moment of leptons $(g-2)$ [21, 22]. Several other processes have not been studied in the context of type III seesaw model such as $\tau \rightarrow \ell P$ and $\tau \rightarrow \ell V$.

In this paper we try to demonstrate that we can have a contribution to lepton flavour violating decays even with one triplet and singlet fermions. The paper is organized as follows: In the next section, we will recall the basic features of type-III seesaw model and discuss the motivations which adds a triplet. In Sec. III we will discuss the constraint on the Yukawa couplings coming from neutrino experiments. Section IV then discusses the analytical LFV τ decay rates with estimates the corresponding LFV observables and conclusions are drawn in section V.

II. Z-MEDIATED LFV IN THE TYPE-III SEESAW MODEL

To study the lepton flavor violating effects in the so-called type-III seesaw models [13–15], we consider the $SU(2)_L$ fermionic triplet with the quantum number of $(1, 3, 0)$ under $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry [14]. For explaining the current data in neutrino physics, model with only one triplet fermion is not sufficient; therefore, more fermionic triplets and/or singlets should be considered. Since our purpose is to illustrate the τ LFV in the framework of type-III seesaw models, for simplicity we focus on the minimal extension of the SM (MSM), i.e. the case with one triplet and one singlet fermions [22]. The detailed analysis in the model with three triplets could be referred to Ref. [29]. Let us describe the model in more detail to identify new tree-level FCNC in the lepton sector, the component

fields of the triplet fermion is chosen to be

$$\mathbf{T} = \begin{pmatrix} T^0/\sqrt{2} & T^- \\ T^+ & T^0/\sqrt{2} \end{pmatrix}. \quad (1)$$

In order to keep invariance under $SU(2)_L$ gauge transformation, we have required the transformation of \mathbf{T} to be $\mathbf{T} \rightarrow U^* \mathbf{T} U^\dagger$. For studying flavor changing effects, we need to understand the gauge couplings of SM leptons and triplet fermion. The Yukawa sector with respect to the gauge symmetry $SU(2)_L \times U(1)_Y$ can be written as [22]

$$\begin{aligned} -\mathcal{L}_Y = & H^\dagger \bar{e}_a Y_E^{ab} L_b + y_T^a \text{Tr}(\bar{T}_L^c L_a H^T) + y_S^a H^T i\tau_2 L_a S \\ & + \frac{1}{2} m_T \text{Tr}(\bar{\mathbf{T}} \mathbf{T}) + \frac{1}{2} m_S S^T C S + h.c. \end{aligned} \quad (2)$$

where $H^T = (\phi^+, \phi^0)$ is the SM Higgs doublet, e_a denotes right-handed lepton and $a(b)$ is the corresponding lepton flavor, $L^T = (\nu, \ell)_L$ is the weak gauge doublet of lepton, Y_E^{ab} and $y_{T,S}^a$ are Yukawa couplings, C is the charge conjugation operator, and $m_{T(S)}$ is the mass of the new stuff in triplet (singlet) fermion.

Similarly, the relevant gauge kinetic terms are written as

$$\mathcal{L}_{\text{kin}} = \bar{L} i \not{D}_2 L + \bar{\ell}_R i \not{D}_1 \ell_R - \text{Tr}[\bar{\mathbf{T}}'_L i \not{D}_3 \mathbf{T}'_L], \quad (3)$$

where $D_{2\mu} = \partial_\mu + ig/2\vec{\tau} \cdot \vec{W}_\mu - ig'/2B_\mu$, $D_{1\mu} = \partial_\mu - ig'B_\mu$ and $D_{3\mu} = \partial_\mu + ig\vec{\tau} \cdot \vec{W}_\mu$ are the covariant derivatives, $\mathbf{T}' = i\tau_2 \mathbf{T}$ and the associated gauge transformation is $\mathbf{T}' \rightarrow U \mathbf{T}' U^\dagger$. Since singlet fermion doesn't couple to gauge boson, we don't show it in Eq. (3). Although charged currents will induce flavor changing effects through box and penguin diagrams, however, due to loop suppression, the Z-mediated LFV induced at tree level will be dominant. Consequently, we focus on the Z-boson related interactions. By Eq. (3), the interactions in weak eigenstates of lepton are found by

$$\mathcal{L}_Z = \frac{g}{2 \cos \theta_W} \bar{\ell} \gamma^\mu (\cos 2\theta_W X_L P_L - 2 \sin^2 \theta_W X_R P_R) \ell Z_\mu \quad (4)$$

with $\ell_L = (e_L, \mu_L, \tau_L, T_L^c)$, $\ell_R = (e_R, \mu_R, \tau_R, T_R^c)$ and

$$X_L = \begin{pmatrix} \mathbb{1}_{3 \times 3} & 0_{3 \times 1} \\ 0_{1 \times 3} & -2 \cos^2 \theta_W / \cos 2\theta_W \end{pmatrix}, \quad X_R = \begin{pmatrix} \mathbb{1}_{3 \times 3} & 0_{3 \times 1} \\ 0_{1 \times 3} & \cos^2 \theta_W / \sin^2 \theta_W \end{pmatrix}. \quad (5)$$

Here, we have taken T^c as the charge-conjugation state of T^+ . Since the couplings of the new charged leptons to Z-boson differ from those of ordinary leptons, we will see that LFV

at tree level will be induced by the misalignment between weak and physical states. To understand the effects of LFV, we first introduce two unitary matrices $V_{L,R}$ that transform the weak states to physical states by $\ell_{L(R)} \rightarrow V_{L(R)}\ell_{L(R)}$. Then the matrices in Eq. (5) become

$$Z_\eta \equiv V_\eta X_\eta V_\eta^\dagger = V_\eta I V_\eta^\dagger + V_\eta (X_\eta - I) V_\eta^\dagger \quad (6)$$

with $\eta = L, R$ and I being the unit matrix. Immediately, we can see that the lepton flavor changing effects are associated with

$$\begin{aligned} Z_{\eta ij} &= \chi_\eta V_{\eta i4} V_{\eta j4}^*, \\ \chi_L &= -2 \cos^2 \theta_W / \cos 2\theta_W - 1, \\ \chi_R &= \cos^2 \theta_W / \sin^2 \theta_W - 1. \end{aligned} \quad (7)$$

To get the information on $V_{\eta i4}$ and $V_{\eta 4i}$, we need to study the detailed mass matrix for charged leptons.

After SSB, the mass matrix for charged lepton could be written as

$$-\mathcal{L}_Y^\ell = \bar{\ell}_R M_\ell \ell_L + h.c. \quad (8)$$

with

$$M_\ell = \begin{pmatrix} (\mathbf{Y}_E)_{3 \times 3} v / \sqrt{2} & | & \mathbf{0}_{3 \times 1} \\ - & - & - & | & - \\ (\mathbf{y}_T^\dagger)_{1 \times 3} v / \sqrt{2} & | & m_T \end{pmatrix}. \quad (9)$$

Moreover, by choosing a suitable basis, indeed Eq. (9) can be further simplified. For instance, using the transformation $\ell_{L(R)} \rightarrow U_{L(R)}\ell_{L(R)}$ with

$$U_{L(R)} = \begin{pmatrix} \bar{U}_{L(R)3 \times 3} & | & \mathbf{0}_{3 \times 1} \\ - & - & - & | & - \\ \mathbf{0}_{1 \times 3} & | & 1 \end{pmatrix},$$

the matrix \mathbf{Y}_E in Eq. (9) can be diagonalized and Eq. (9) becomes

$$M_\ell = \begin{pmatrix} (\mathbf{m}_E)_{3 \times 3} & | & \mathbf{0}_{3 \times 1} \\ - & - & - & | & - \\ (\mathbf{h}_T^\dagger)_{1 \times 3} & | & m_T \end{pmatrix}, \quad (10)$$

where $\text{diag}(\mathbf{m}_E) = \text{diag}(\bar{U}_R(\mathbf{Y}_E v/\sqrt{2})\bar{U}_L^\dagger) = (m_e, m_\mu, m_\tau)$ and $\mathbf{h}_T = \bar{U}_L \mathbf{Y}_T v/\sqrt{2}$. Since M_ℓ still has off-diagonal elements, clearly $m_{e,\mu,\tau}$ are not physical eigenstates. In addition, from Eq. (10) one can expect that the lepton flavor violating effects will be associated with \mathbf{h}_T . To get the physical states, we use $V_{R,L}$ introduced early to diagonalize the mass matrix of lepton, i.e. $M_\ell^{\text{dia}} = V_R M_\ell V_L^\dagger$. The individual information on V_L and V_R can be obtained by

$$\begin{aligned} M_\ell^{\text{dia}\dagger} M_\ell^{\text{dia}} &= V_L M_\ell^\dagger M_\ell V_L^\dagger, \\ M_\ell^{\text{dia}} M_\ell^{\text{dia}\dagger} &= V_R M_\ell M_\ell^\dagger V_R^\dagger \end{aligned} \quad (11)$$

with

$$M_\ell^\dagger M_\ell = \left(\begin{array}{c|c} \mathbf{m}_E^\dagger \mathbf{m}_E + \mathbf{h}_T \mathbf{h}_T^\dagger & \mathbf{h}_T m_T \\ \hline - & - \\ \hline m_T \mathbf{h}_T^\dagger & m_T^2 \end{array} \right), \quad M_\ell M_\ell^\dagger = \left(\begin{array}{c|c} \mathbf{m}_E \mathbf{m}_E^\dagger & \mathbf{m}_E \mathbf{h}_T \\ \hline - & - \\ \hline \mathbf{h}_T^\dagger \mathbf{m}_E^\dagger & m_T^2 + \mathbf{h}_T^\dagger \mathbf{h}_T \end{array} \right).$$

Clearly, V_L and V_R are the unitary matrices to diagonalize the matrix $M_\ell^\dagger M_\ell$ and $M_\ell M_\ell^\dagger$, respectively. Expectably, the off-diagonal elements of flavor mixing matrices will be associated with $(\mathbf{m}_{Eii} \mathbf{h}_{Ti})$ and $m_T \mathbf{h}_{Ti}$ which reflect the mixture of ordinary quarks and triplet fermion. Although in general the 4×4 matrices will be complicated and unknown, however, since the introduced triplet fermions are much heavier than SM leptons, i.e. $m_T \gg \mathbf{m}_{Eii}, \mathbf{h}_{Ti} \sim v$, for a good approximation we can expand $V_{L(R)}$ to be $V_{L(R)} \approx \mathbb{1}_{4 \times 4} + \Delta_{L(R)}$ where $\Delta_{L(R)}$ is regarded as $O(h_{Ti}/m_T)[O(m_{Eii} h_{Ti}/m_T^2)]$. Comparing with Eq. (7), we see $V_{\eta i 4(4i)} \approx \Delta_{\eta i 4(4i)}$. From Eq. (11), we can derive the leading order for flavor mixing as

$$\Delta_{Li4} \approx \Delta_{L4i}^* \approx -\frac{m_T h_{Ti}}{m_T^2 - m_{Ei}^2 - |h_{Ti}|^2}, \quad (12)$$

$$\Delta_{Ri4} \approx -\Delta_{R4i}^* \approx -\frac{m_{Ei} h_{Ti}}{m_T^2 + \mathbf{h}_T^\dagger \mathbf{h}_T - m_{Ei}^2}. \quad (13)$$

Since $m_T^2 \gg m_T h_{Ti} \gg m_{Ei} h_{Ti}$, it is clear that the effects of $\Delta_{Ri4(4i)}$ are negligible. Hence, the significant LFV in type-III seesaw model is only associated with left-handed neutral currents.

III. CONSTRAINTS ON THE PHYSICAL PARAMETERS

In this section, we discuss the constraints coming from the neutrino experiments on the relevant Yukawa couplings y_T^a . After spontaneous symmetry breaking (SSB), where the

Higgs field is driven to obtain the VEV i.e. $\langle \phi^0 \rangle = v/\sqrt{2}$, the light neutrino mass matrix is given by

$$(m^\nu)^{ab} = -\frac{v^2}{2} \left(\frac{y_T^a y_T^b}{m_T} + \frac{y_S^a y_S^b}{m_S} \right) \quad (14)$$

The unitary PMNS matrix that diagonalizes the neutrino mass matrix Eq. (14) is given by

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \text{diag}(1, e^{i\Phi}, 1). \quad (15)$$

where, $s_{ij} = \sin \theta_{ij}$, $c_{ij} = \cos \theta_{ij}$ ($i, j = 1, 2, 3$), δ is the CP-violating Dirac phase and Φ denotes the Majorana phase. The experimental constraints on the neutrino masses and mixing parameters, at 2σ level [24, 25] are

$$7.3 \times 10^{-5} \text{eV}^2 < \Delta m_S^2 < 8.1 \times 10^{-5} \text{eV}^2 \quad (16)$$

$$2.1 \times 10^{-3} \text{eV}^2 < |\Delta m_A^2| < 2.7 \times 10^{-3} \text{eV}^2 \quad (17)$$

$$0.28 < \sin^2 \theta_{12} < 0.37, \quad 0.38 < \sin^2 \theta_{23} < 0.68, \quad \sin^2 \theta_{13} < 0.033 \quad (18)$$

In this section, we focus mainly on the case of Normal Hierarchy (NH, $m_1^\nu = 0$), and Inverted Hierarchy (IH, $m_3^\nu = 0$) neglecting the Majorana phase. Using the above experimental constraints, the neutrino masses are given by [24, 25]

$$m_2^\nu = \sqrt{\Delta m_S^2}, \quad m_3^\nu = \sqrt{\Delta m_S^2 + \Delta m_A^2} \quad (19)$$

in the case of NH, and

$$m_1^\nu = \sqrt{\Delta m_A^2 - \Delta m_S^2}, \quad m_2^\nu = \sqrt{\Delta m_A^2} \quad (20)$$

in the case of IH. The constraints on the neutrino mass matrix elements directly translate into the physical Yukawa couplings y_T^a . Using Casas-Ibarra parametrization [26], one can find a formal solution for the Yukawa couplings y_T^a and y_S^a can be expressed as

$$y_T^a = -i \frac{\sqrt{2} m_T}{v} \left(\sqrt{m_2^\nu} \cos z U_{a2}^* + \sqrt{m_3^\nu} \sin z U_{a3}^* \right) \quad (21)$$

$$y_S^a = -i \frac{\sqrt{2} m_S}{v} \left(-\sqrt{m_2^\nu} \sin z U_{a2}^* + \sqrt{m_3^\nu} \cos z U_{a3}^* \right) \quad (22)$$

for NH, and

$$y_T^a = -i \frac{\sqrt{2m_T}}{v} \left(\sqrt{m_1^\nu} \cos z U_{a1}^* + \sqrt{m_2^\nu} \sin z U_{a2}^* \right) \quad (23)$$

$$y_S^a = -i \frac{\sqrt{2m_S}}{v} \left(-\sqrt{m_1^\nu} \sin z U_{a1}^* + \sqrt{m_2^\nu} \cos z U_{a2}^* \right) \quad (24)$$

for IH, where $a = 1, 2, 3$ and z is a complex parameter. In order to study the effect of the z parameter, we show in Fig.1 the relative size of the Yukawa couplings y_T^a as a function of $\text{Im}(z)$ for IH (left panel) and NH (right panel) with fixed $m_T = 1$ TeV, when ignoring the influence of the Majorana phase Φ . As we can see from both panels, for large $\text{Im}(z)$, the Yukawa couplings remain large and even $\mathcal{O}(1)$, which can be understood from Eqs. (21)-(24) where y_T^a are proportional to $e^{\text{Im}(z)}$. This allows to account for the experimental values of neutrino masses without fine-tuning the Yukawa couplings due to a cancellations in combination of them, the observable effects are then possible.

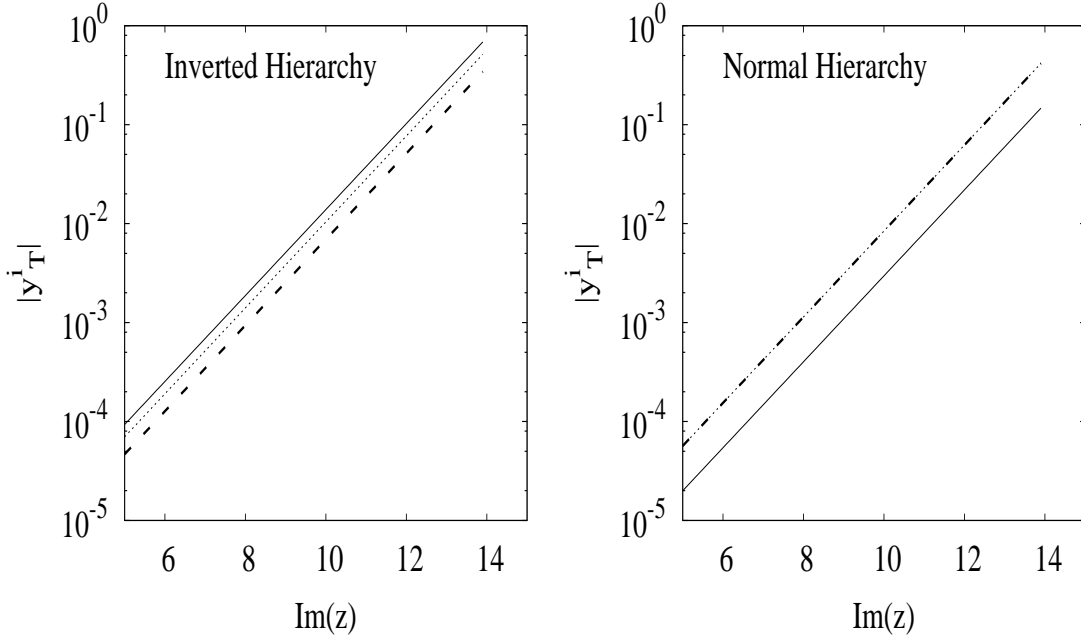


FIG. 1: The absolute value of Yukawa couplings y_T^a as a function of $\text{Im}(z)$ for IH (left) and NH (right), with $m_T = 1$ TeV and $\Phi = 0$, in which the solid, dashed and dotted lines represent $a=1, 2$, and 3, respectively.

IV. DECAY RATES FOR $\tau \rightarrow \ell M$ AND $\tau \rightarrow 3\ell$

Based on previous analysis, now we can study the LFV of the type-III seesaw model on semileptonic $\tau \rightarrow \ell M$ with $M = (P, V)$ and leptonic $\tau \rightarrow 3\ell$ decays. According to the interactions in Eq. (4) and Z couplings in the SM, the relevant effective Hamiltonian for τ flavor changing decays can be written as

$$\mathcal{H} = \frac{4G_F}{\sqrt{2}} \cos 2\theta_W Z_{Li3} \left(g_L^f \bar{f} \gamma^\mu P_L f \bar{\ell}_i \gamma_\mu P_L \tau + g_R^f \bar{f} \gamma^\mu P_R f \bar{\ell}_i \gamma_\mu P_L \tau \right) \quad (25)$$

with

$$\begin{aligned} g_L^f &= I_{3f} - Q_f \sin^2 \theta_W, \\ g_R^f &= -Q_f \sin^2 \theta_W, \end{aligned} \quad (26)$$

where we have used the equalities $\cos \theta_W = m_W/m_Z$ and $g^2/8m_W^2 = G_F/\sqrt{2}$, f could be leptons and quarks, and I_{3f} and Q_f denote the third component of weak isospin and electric charge of the particle f , respectively. Since semileptonic τ decays are associated with meson production where the decay constants of nonperturbative hadronic effects involve, for dealing with the hadronic effects, as usual the decay constants of pseudo-scalar (P) and vector (V) mesons are defined as

$$\begin{aligned} \langle 0 | \bar{q} \gamma^\mu \gamma_5 q | P(p) \rangle &= i f_P p^\mu, \\ \langle 0 | \bar{q} \gamma^\mu q | V(p) \rangle &= i m_V f_V \varepsilon_V^\mu \end{aligned} \quad (27)$$

with ε_V^μ being the polarization vector of vector meson. Moreover, for the modes associated with η and η' mesons, we employ the quark-flavor scheme in which η and η' physical states are described by [27, 28]

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} \quad (28)$$

with ϕ being the mixing angle, $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$. Accordingly, the decay constant of $\eta^{(\prime)}$ associated with $\bar{q} \gamma^\mu \gamma_5 q$ ($q = u, d$) current is given by $f_{\eta^{(\prime)}} = \cos \phi (\sin \phi) f_{\eta_q}$. Consequently, for $\tau \rightarrow \ell_i P$ process, the decay amplitude can be summarized by

$$\langle P \ell_i | \mathcal{H} | \tau \rangle = \sqrt{2} G_F f_P m_\tau \cos 2\theta_W Y_P Z_{Li3} \bar{\ell}_i P_R \tau, \quad (29)$$

while for $\tau \rightarrow \ell_i V$ decay, it is

$$\langle V \ell_i | \mathcal{H} | \tau \rangle = \sqrt{2} G_F f_V m_V \cos 2\theta_W Y_V Z_{Li3} \bar{\ell}_i \not{e}_V P_L \tau, \quad (30)$$

where $f_{\eta'} = \sin \phi (\cos \phi) f_{\eta_s}$,

$$\begin{aligned} Y_{\pi^0} &= -\frac{1}{\sqrt{2}}, & Y_\eta &= -\frac{1}{2}, & Y_{\eta'} &= \frac{1}{2}, \\ Y_{\rho^0} &= \frac{\cos 2\theta_W}{\sqrt{2}}, & Y_\omega &= -\frac{2}{3\sqrt{2}} \sin^2 \theta_W, & Y_\phi &= -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W. \end{aligned} \quad (31)$$

Due to $m_{e,\mu} \ll m_\tau$, we have neglected the masses of light leptons. Hence, the BRs for semileptonic τ LFV are found to be

$$\mathcal{B}(\tau \rightarrow \ell_i P) = \frac{G_F^2}{16\pi\Gamma_\tau} f_P^2 m_\tau^3 \cos^2 2\theta_W Y_P^2 |Z_{Li3}|^2 \left(1 - \frac{m_P^2}{m_\tau^2}\right)^2, \quad (32)$$

$$\mathcal{B}(\tau \rightarrow \ell_i V) = \frac{G_F^2}{16\pi\Gamma_\tau} f_V^2 m_\tau^3 \cos^2 2\theta_W Y_V^2 |Z_{Li3}|^2 \left(1 - \frac{m_V^2}{m_\tau^2}\right)^2 \left(1 + 2\frac{m_V^2}{m_\tau^2}\right). \quad (33)$$

For leptonic $\tau \rightarrow 3\ell$ decays, although there involve no hadronic effects, however, they are three body decays and have more complicated phase space. To simplify the formulation, we neglect the effects of light lepton masses. Hence, using the interactions in Eq. (4), the BR for $\tau \rightarrow 3\ell$ is given by

$$\mathcal{B}(\tau \rightarrow \ell_i \ell \bar{\ell}) = \cos^2 2\theta_W |Z_{Li3}|^2 (\zeta |g_L^\ell|^2 + |g_R^\ell|^2) \mathcal{B}(\tau \rightarrow \ell \nu_\tau \bar{\nu}_\ell) \quad (34)$$

where $\zeta = 2$ for $\ell_i = \ell$ and $\zeta = 1$ for $\ell_i \neq \ell$.

After introducing the contributions of Z-mediated LFV in the type-III seesaw model, in order to constrain the free parameters of Z_{Li3} , we have to find out the possible strict limits. As known that the Z-mediated effects at tree level in the SM are flavor conserved, intuitively the flavor violating decays $Z \rightarrow \ell_i \bar{\ell}_j$ with $i \neq j$, $\tau \rightarrow \ell(P, V)$ and $\tau \rightarrow 3\ell$ etc could give strong constraints on the unknown parameters. Therefore, by examining the relation of the BR and the associated parameter, one can easily find $|Z_{Lij}| = |\Delta_{Li4} \Delta_{Lj4}^*| \propto \sqrt{BR}$. By taking $\mathcal{B}(Z \rightarrow \ell_i \bar{\ell}_j) \sim 10^{-6}$ and $\mathcal{B}(\tau \rightarrow [\ell(P, V), 3\ell]) \sim 10^{-7}$, we see $|\Delta_{Li4} \Delta_{Lj4}^*| \propto 10^{-3}$ and $|\Delta_{Li4} \Delta_{Lj4}^*| \propto 10^{-4}$, respectively. However, if we further think, the current high precision measurements of $Z \rightarrow \ell_i \bar{\ell}_i$ processes in fact will provide more severe limits. Roughly speaking, the key reason can be understood by which the allowed range for new physics is directly governed by small errors of data for $Z \rightarrow \ell_i \bar{\ell}_i$, *i.e.* $|\Delta_{Li4}|^2 \propto \Delta\Gamma/\Gamma_Z \equiv [\Gamma^{\text{exp}}(Z \rightarrow$

$\ell_i \bar{\ell}_i) - \Gamma^{\text{SM}}(Z \rightarrow \ell_i \bar{\ell}_i)]/\Gamma_Z \sim 10^{-5}$ [23]. Although the constraint in the real situation will depend on the detailed characters of the process, however, we will adopt current data of $Z \rightarrow \ell_i \bar{\ell}_i$ with 1σ errors as the inputs and the BRs for $\tau \rightarrow \ell(P, V)$ and $\tau \rightarrow 3\ell$ decays are our predictions.

In terms of the Z couplings in Eq. (4), the BR for $Z \rightarrow \ell_i \bar{\ell}_i$ decay with new effects can be formulated as

$$\mathcal{B}(Z \rightarrow \ell_i \bar{\ell}_i) = \mathcal{B}^{\text{SM}}(Z \rightarrow \ell_i \bar{\ell}_i) + \xi_Z |\Delta_{Li4}|^2 \quad (35)$$

with

$$\begin{aligned} \mathcal{B}^{\text{SM}}(Z \rightarrow \ell_i \bar{\ell}_i) &= \frac{G_F m_Z^3}{3\sqrt{2}\pi\Gamma_Z} \left[\left(\frac{\cos 2\theta_W}{2} \right)^2 + \sin^4 \theta_W \right], \\ \xi_Z &= \frac{m_Z^3 G_F}{6\sqrt{2}\pi\Gamma_Z} \cos^2 2\theta_W \chi_L. \end{aligned} \quad (36)$$

Due to $m_\ell \ll m_Z$, here we have dropped the mass of lepton. In addition, we also neglected the terms that power in free parameter is higher than $|\Delta_{Li4}|^2$. As a result, the allowed range for unknown parameter can be bounded by

$$|\Delta_{Li4}|^2 = \frac{\Delta \mathcal{B}_i}{\xi_Z} = \frac{1}{\xi_Z} [\mathcal{B}^{\text{exp}}(Z \rightarrow \ell_i \bar{\ell}_i) - \mathcal{B}^{\text{SM}}(Z \rightarrow \ell_i \bar{\ell}_i)]. \quad (37)$$

By using $G_F = 1.16634 \times 10^{-5} \text{ GeV}^{-2}$, $\sin^2 \theta_W = 0.223$ and $\Delta \mathcal{B}_{e, \mu, \tau} = (4, 7, 8) \times 10^{-5}$ where the values are taken from 1σ errors of data for $Z \rightarrow \ell_i \bar{\ell}_i$ [23], the upper limit on $|\Delta_{Li4}|$ is given in Table I. Thus, based on Eqs. (32) and (33), the upper limits on the BRs for

TABLE I: Upper limit on Δ_{Li4} with 1σ errors of $\mathcal{B}(Z \rightarrow \ell_i \bar{\ell}_i)$.

Mode	$e^- e^+$	$\mu^- \mu^+$	$\tau^- \tau^+$
$ \Delta_{Li4} $	0.016	0.021	0.023

$\tau \rightarrow \ell(\pi^0, \eta, \eta')$ and $\tau \rightarrow \ell(\rho^0, \omega, \phi)$ are shown in Tables II and III, respectively. Here, we have used the hadronic values as

$$\begin{aligned} f_\pi &= 0.13, & f_\eta &= 0.11, & f'_\eta &= 0.135, \\ f_\rho &= 0.216, & f_\omega &= 0.187, & f_\phi &= 0.237 \end{aligned} \quad (38)$$

in unites of GeV. From the values in the tables, we see clearly that the upper limits on the BRs for $\tau \rightarrow \ell(\pi^0, \rho^0, \phi)$ could be $\mathcal{O}(10^{-8})$ and the order in size is $\mathcal{B}(\tau \rightarrow \ell\pi^0) > \mathcal{B}(\tau \rightarrow$

$\ell\rho^0) > \mathcal{B}(\tau \rightarrow \ell\phi^0)$. Therefore, the semileptonic $\tau \rightarrow \ell(\pi^0, \rho^0)$ decays could be the good candidates to probe the Z-mediated τ LFV.

TABLE II: Upper limits on the BRs (in units of 10^{-8}) for $\tau \rightarrow \ell(\pi^0, \eta, \eta')$ decays with 1σ errors of $\mathcal{B}(Z \rightarrow \ell_i \bar{\ell}_i)$ as the constraints.

Mode	$\tau \rightarrow (e, \mu)\pi^0$	$\tau \rightarrow (e, \mu)\eta$	$\tau \rightarrow (e, \mu)\eta'$
Current limit	(8.0, 11)	(9.2, 6.5)	(16, 13)
This work	(2.8, 5.0)	(0.6, 1.0)	(0.4, 0.7)

TABLE III: Upper limits on the BRs (in units of 10^{-8}) for $\tau \rightarrow \ell(\rho^0, \omega, \phi)$ decays with 1σ errors of $\mathcal{B}(Z \rightarrow \ell_i \bar{\ell}_i)$ as the constraints.

Mode	$\tau \rightarrow (e, \mu)\rho^0$	$\tau \rightarrow (e, \mu)\omega$	$\tau \rightarrow (e, \mu)\phi$
Current limit	(6.3, 6.8)	(11, 8.9)	(7.3, 13)
This work	(1.7, 3.0)	(0.09, 0.2)	(1.1, 1.7)

With the same constraints shown in the Table I, the values of BRs for leptonic τ decays formulated by Eq. (34) are presented in Table IV. It is clear that the BRs for all $\tau \rightarrow 3\ell$ decays are of order 10^{-8} and the predictions are close to each other. Furthermore, from the Table IV, one can find that the value of $\mathcal{B}(\tau \rightarrow 3\mu)$ is a little bit larger than current experimental upper limit. It seems that $\tau \rightarrow 3\mu$ provides the most strictest constraint on the free parameters. However, by reexamining the constraints of $Z \rightarrow \ell_i \bar{\ell}_i$, we find that the reverse situation is arisen from the errors of $Z \rightarrow (\tau^-\tau^+, \mu^-\mu^+)$ being larger than that of $Z \rightarrow e^-e^+$, i.e. $\Delta\mathcal{B}_\tau \sim \Delta\mathcal{B}_\mu > \Delta\mathcal{B}_e$. If we adopt 3σ of the world average $\mathcal{B}(Z \rightarrow \ell_i \bar{\ell}_i) = (3.3658 \pm 0.0023)\%$ for $\ell = e, \mu$ and τ , the new upper limits on the BRs for semileptonic and leptonic decays are found to be

$$\begin{aligned}
\mathcal{B}[\tau \rightarrow \ell(\pi^0, \eta, \eta')] &< (4.2, 0.8, 0.6) \times 10^{-8}, \\
\mathcal{B}[\tau \rightarrow \ell(\rho^0, \omega, \phi)] &< (2.5, 0.1, 1.6) \times 10^{-8}, \\
\mathcal{B}[\tau \rightarrow (3\ell, \mu e^- e^+, e\mu^- \mu^+)] &< (3.1, 2.0, 2.0) \times 10^{-8}.
\end{aligned} \tag{39}$$

Clearly, precision measurements of $Z \rightarrow \ell_i \bar{\ell}_i$ play an essential role on the constraints.

TABLE IV: Upper limits on the BRs (in units of 10^{-8}) for $\tau \rightarrow 3\ell$ decays with 1σ errors of $\mathcal{B}(Z \rightarrow \ell_i \bar{\ell}_i)$ as the constraints.

Mode	$\tau \rightarrow 3e$	$\tau \rightarrow 3\mu$	$\tau \rightarrow \mu e^- e^+$	$\tau \rightarrow e \mu^- \mu^+$
Current limit	3.6	3.2	2.7	3.7
This work	2.1	3.7	2.3	1.3

V. CONCLUSIONS

We have investigated the lepton flavor violating effects in the framework of type-III seesaw model by extending the SM with one $SU(2)_L$ triplet and singlet fermions. Due to the difference in weak charges between new and ordinary leptons, intriguingly Z-mediated LFV is generated at tree level. Moreover, it is found that the significant effects only occur in the left-handed leptons. Although LFV could be induced by charged currents through one-loop, however, comparing with tree contributions, they are subleading effects and neglected in our analysis. To illustrate the novel effects, we study the semileptonic $\tau \rightarrow \ell M$ and leptonic $\tau \rightarrow 3\ell$ decays. For numerical calculations, we find that the precision measurements of $Z \rightarrow \ell_i \bar{\ell}_i$ play an important role on the constraints of the free parameters. Furthermore, we find that the upper limits on the BRs for $\tau \rightarrow \ell(\pi^0, \rho^0, \phi)$ and $\tau \rightarrow 3\ell$ could reach $\mathcal{O}(10^{-8})$ in the model under discussion.

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